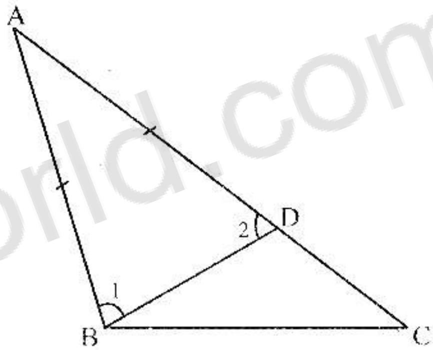
Unit 05

FUNDAMENTAL CONCEPTS OF GEOMETRY

Theorem 1

If two sides of a triangle are unequal in length, the measure of the angle opposite to the longer side is greater the shorter side.



In ΔABC, $m\overline{AC} > \overline{AB}$

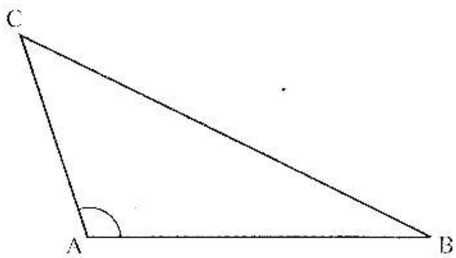
$m\angle ABC > m\angle ACB$

On $\overline{AC}$ take $\overline{AD} \cong \overline{AB}$ Join B to D so that ΔADB is an isosceles triangle.

Statements	Reasons
In ΔABD	
$m\angle 1 = m\angle 2$ (i)	Angles opposite to congruent sides.
In ΔBCD	
$m\angle 2 > m\angle ACB$ (ii)	An exterior angle of a triangle is greater than every non adjacent angle.
$\therefore m\angle 1 > m\angle ACB$ (iii)	By (i) and (ii)
Now	
$m\angle ABC = m\angle 1 + m\angle DBC$	Postulate of addition of measure of angles
$\therefore m\angle ABC > m\angle 1$ (iv)	
$m\angle ABC > m\angle 1$ $m\angle ACB$	By (iii) and (iv)
or $m\angle ABC > m\angle ACB$	Transitive property of inequality of real numbers.

Theorem 1(a) (Converse of Theorem 1)

If two angles of a triangle are unequal in measure, the side opposite to greater angle is longer than the side opposite to the smaller angle.



In Δ ABC

$m\angle A > m\angle B$

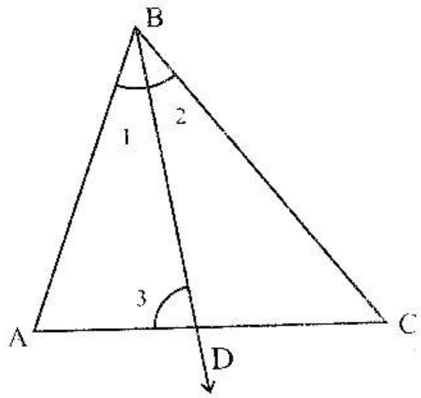
$m\overline{BC} > \overline{AC}$

Proof:

Statements	Reasons
If $m\overline{BC} \not> m\overline{AC}$, then either (i) $m\overline{BC} = m\overline{AC}$ or (ii) $m\overline{BC} < m\overline{AC}$	Tracheotomy property of real numbers
From (i) if $m\overline{BC} = m\overline{AC}$, then $m\angle A = m\angle B$	Angles opposite to congruent sides are congruent
Which is not possible	Contrary to what is given.
From (ii) if $m\overline{BC} < m\overline{AC}$, then $m\angle A < m\angle B$	The angle opposite to longer side is greater than angle opposite to smaller side.
This is also not possible being contrary to what is given	
$\therefore$ $m\overline{BC} \neq m\overline{AC}$	
And $m\overline{BC} \not< m\overline{AC}$ Hence $m\overline{BC} > m\overline{AC}$	Trichotomy property of real numbers

Theorem 2

The sum of measures of any two sides of a triangle is greater than the measure of the third side.



Given: A triangle ABC

To Prove:

(i) $m\overline{AB} + m\overline{BC} > m\overline{AC}$

(ii) $m\overline{AC} + m\overline{AB} > m\overline{BC}$

(iii) $m\overline{AC} + m\overline{BC} > m\overline{AB}$

Construction:

Draw the bisector of $\angle$ B to meet the side $\overline{AC}$ at the point D.

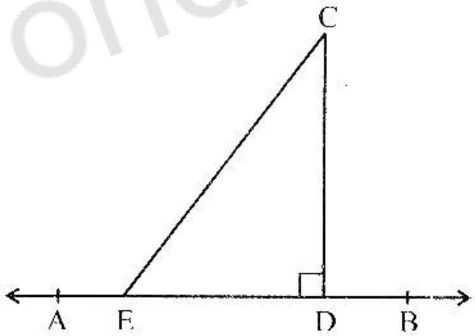
Proof:

Statements	Reasons
In Δ CBD $m\angle 3 > m\angle 2$ (i)	Exterior angle is greater than non adjacent interior angle.
$m\angle 2 = m\angle 1$ (ii)	Construction
$\therefore$ $m\angle 3 > m\angle 1$	From (i) and (ii)

and $m\overline{AB} > m\overline{AD}$ (iii) Similarly $m\overline{BC} > m\overline{DC}$ (iv)	Construction In ΔABD the side that opposite to the large angle is greater than of the side opposite to the smaller angle.
$m\overline{AB} + m\overline{BC} > m\overline{AD} + m\overline{DC}$ $m\overline{AB} + m\overline{BC} > m\overline{AC}$	Adding (iii) and (iv) $\therefore\ m\overline{AD} + m\overline{DC} = m\overline{AC}$
Similarly by drawing angle bisectors of $\angle A$ and $\angle C$ it can be proved that $m\overline{AC} + m\overline{AB} > m\overline{BC}$ and $m\overline{AC} + m\overline{BC} > m\overline{AB}$	

Theorem 3

From a point, outside a line, the perpendicular is the shortest distance from the point to the line.



Given:

A line $\overleftrightarrow{AB}$ and a point C (not lying on $\overleftrightarrow{AB}$) and a point D on $\overleftrightarrow{AB}$ such that $\overline{CD}$ is perpendicular to $\overleftrightarrow{AB}$.

To Prove:

$m\overline{CD}$ is the shortest distance from point C to the line $\overleftrightarrow{AB}$.

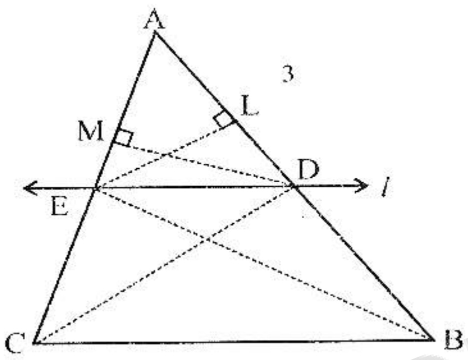
Construction: Take a point E on $\overleftrightarrow{AB}$ Join C and E to get a ΔCDE.

Proof:

Statements	Reasons
In ΔCDE $m\angle CDB > m\angle CED$	An exterior angle of a triangle is greater than every non adjacent interior angle.
But $m\angle CDB = m\angle CDE$	Supplement of right angle
$\therefore m\angle CDE > m\angle CED$	
or $m\angle CED < m\angle CDE$	Reflexive property of inequality.
or $m\overline{CD} < m\overline{CE}$	Side opposite to greater angle is greater.
Bur E was any point on $\overleftrightarrow{AB}$ Hence $m\overline{CD}$ is the shortest distance from C to $\overleftrightarrow{AB}$.	

Theorem 4

A line parallel to one side of a triangle and intersecting the others two sides divides them proportionally.



Given:

In ΔABC, the line l is intersecting the sides $\overline{AC}$ and $\overline{AB}$ at points E and D respectively such that $\overline{ED} \parallel \overline{CB}$

To Prove:

$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$

Construction:

Join B to E and C to D and draw $\overline{DM}$ and $\overline{EL}$ perpendiculars on $\overline{AC}$ and $\overline{AB}$ respectively from D and E

Proof:

Statements	Reasons
In triangle BED and AED, $\overline{EL}$ is the common perpendicular	
$\Delta BED = \frac{1}{2} \times m\overline{BD} \times m\overline{EL}$ (i)	Area of triangle = $\frac{1}{2}$ (base x height)
$\Delta AED = \frac{1}{2} \times m\overline{BD} \times m\overline{EL}$ (ii)	
$\frac{\Delta BED}{\Delta AED} = \frac{m\overline{BD}}{m\overline{AD}}$ (a)	Dividing (i) by (ii)
Similarly $\frac{\Delta CDE}{\Delta ADE} = \frac{m\overline{EC}}{m\overline{AE}}$ (b)	
But $\Delta BED = \Delta CDE$	Areas of triangles with common base and same altitudes are equal. $\overline{ED} \parallel \overline{CB}$ given, so altitudes are equal
$\therefore$ From (a) and (b), we have $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	
$\therefore m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$	

Note:

i. $\dfrac{m\overline{BD}}{m\overline{AB}} = \dfrac{m\overline{CE}}{m\overline{AC}}$ and $\dfrac{m\overline{AD}}{m\overline{AB}} = \dfrac{m\overline{AE}}{m\overline{AC}}$

ii. If $\dfrac{m\overline{AD}}{m\overline{AB}} = \dfrac{m\overline{AE}}{m\overline{AC}}$, then $\overline{DE} \parallel \overline{BC}$

iii. If $\dfrac{m\overline{AB}}{m\overline{BD}} = \dfrac{m\overline{AC}}{m\overline{EC}}$, then $\overline{DE} \parallel \overline{BC}$

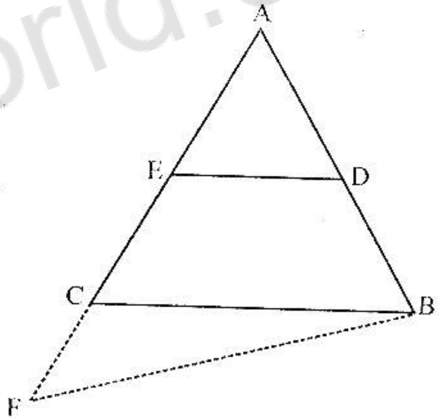
iv. Two points determine a line and three non collinear points determine a plane

v. A line segments has exactly one midpoint.

vi. If two intersecting lines from equal adjacent angles, the lines are perpendicular.

Theorem 4a (Converse of Theorem 4)

If a line segment intersects the two sides of a triangle in the same ratio, then it is parallel to the third side.



Given:

In triangle ABC, $\overline{ED}$ intersects $\overline{AB}$ and $\overline{AC}$ such that

$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$

To Prove:

$\overline{ED} \parallel \overline{CB}$

Construction:

If $\overline{ED} \nparallel \overline{CB}$, then draw $\overline{BF} \parallel \overline{DE}$ to meet $\overline{AC}$ produced at F.

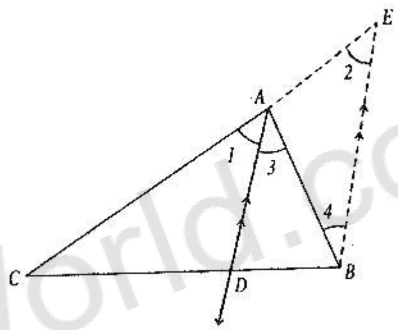
Proof:

Statements	Reasons
In ΔABF	
$\overline{DE} \parallel \overline{BF}$	Construction
$\therefore \dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EF}}$ (i)	A line parallel to one side of a triangle divides the other two sides proportionally
But $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$ (ii)	Given
$\dfrac{m\overline{AE}}{m\overline{EF}} = \dfrac{m\overline{AE}}{m\overline{EC}}$	

or $m\overline{EF} = m\overline{EC}$ Which is possible only if point F is coincident with C. $\therefore$ Our supposition is wrong Hence $\overline{ED} \parallel \overline{CB}$	From (i) and (ii) Property of real numbers.

Theorem 5

The internal bisector of an angle of a triangle divides the sides opposite to it in the ratio of the lengths of sides containing the angle.



Given:

In ΔABC internal angle bisector of angle A meets $\overline{CB}$ at the point D.

To Prove:

$m\overline{BD} : m\overline{DC} = m\overline{AB} : \overline{AC}$

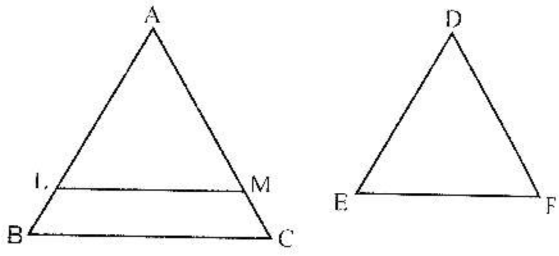
Construction: Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet $\overline{CA}$ produced at E.

Proof:

Statements	Reasons
$\therefore$ $\overline{AD} \parallel \overline{ED}$ and $\overline{EC}$ intersects there at A and E, so $m\angle 1 = m\angle 2$ (i)	Construction corresponding angles
Again $\overline{AD} \parallel \overline{EB}$ and $\overline{AB}$ intersects them so $m\angle 3 = m\angle 4$ (ii)	Alternate angle
But $m\angle 1 = m\angle 3$	Construction (given)
$\therefore m\angle 2 = m\angle 4$	
And $\overline{AE} \cong \overline{AB}$	
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore \dfrac{m\overline{BD}}{m\overline{DC}} = \dfrac{m\overline{EA}}{m\overline{AC}}$	
or $\dfrac{m\overline{BD}}{m\overline{DC}} = \dfrac{m\overline{AB}}{m\overline{AC}}$	$\therefore m\overline{EA} = m\overline{AB}$ (Proved)
$\therefore m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$	

Theorem 6

If two triangles are similar, then measures of their corresponding sides are proportional.



Given:

$\Delta ABC \sim \Delta DEF$

i.e. $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$

To Prove:

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

Construction:

(a). Suppose that $m\overline{AB} > m\overline{DE}$ (b). $m\overline{AB} \leq m\overline{DE}$

On $\overline{AB}$ take a point L such that $m\overline{AL} = m\overline{DE}$

On $\overline{AC}$ take a point M such that $m\overline{AM} = m\overline{DF}$. Join L and M by the line segment LM.

Proof:

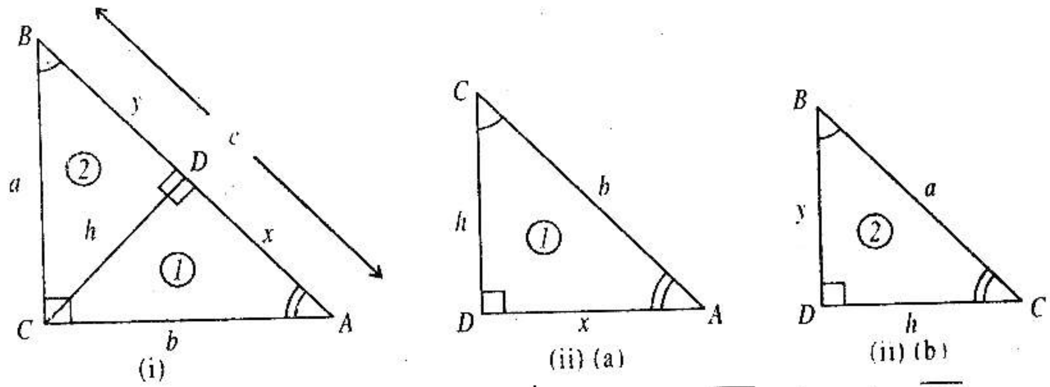
Statements	Reasons
(a). In $\Delta ALM \leftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\Delta ALM \cong \Delta DEF$	S.A.S. postulate
and $\angle L \cong \angle E$, $\angle M \cong \angle F$	Corresponding angles of congruent angles
Now $\angle E \cong \angle B$ and $\angle F \cong \angle C$	Given
$\therefore \angle L \cong \angle B$, $\angle M \cong \angle C$	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal
Hence $\dfrac{m\overline{AL}}{m\overline{AB}} = \dfrac{m\overline{AM}}{m\overline{AC}}$ (i)	By theorem 4
or $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{DF}}{m\overline{AC}}$	$m\overline{AL} = m\overline{DE}$ (construction) $m\overline{AM} = m\overline{DF}$
Similarly by intercepting segments on $\overline{BA}$ and $\overline{BC}$, we can prove that $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{EF}}{m\overline{BC}}$ (ii)	

Thus $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}} = \frac{m\overline{EF}}{m\overline{BC}}$	by (i) and (ii)
Thus $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$	by taking reciprocals.
(b). If $m\overline{AB} < m\overline{DE}$. It can similarly be proved by taking intercepts on the sides of ΔDEF.	
If $m\overline{AB} = m\overline{DE}$	
Then in $\Delta ABC \leftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\angle B \cong \angle E$	Given
and $\overline{AB} \cong \overline{DE}$	
so $\Delta ABC \cong \Delta DEF$	$ASA \cong ASA$
thus $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}} = 1$	$\overline{AC} \cong \overline{DF}$
Thus result is true for all the cases	$\overline{BC} \cong \overline{EF}$

Theorem 7

Pythagoras Theorem

In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the others two sides.



Given: ΔACB is a right angled triangle in which $m\angle C = 90^\circ$ and $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AB} = c$

To Prove: $c^2 = a^2 + b^2$

Construction: Draw $\overline{CD}$ perpendicular from C on $\overline{AB}$. Let $m\overline{CD} = h$. $m\overline{AD} = x$ and $m\overline{BD} = y$. Line segment CD splits ΔABC into two triangles ADC and BDC which are separately shown in the figures ii(a) and ii(b) respectively.

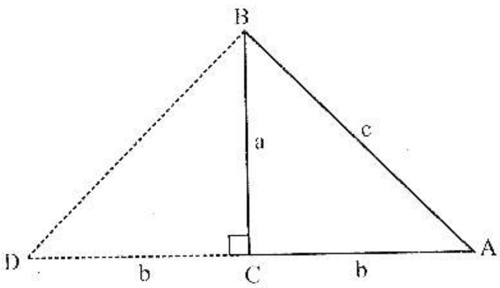
Proof:

Statements	Reasons
(a). In $\Delta ADC \leftrightarrow \Delta ACB$	
$\angle A \cong \angle A$	common-self congruent
$\angle ADC \cong \angle ACB$	Construction given both measure 90°
$\angle C \cong \angle B$	$\angle C$ and $\angle B$ complements of $\angle A$
$\therefore \Delta ADC \sim \Delta ACB$	Congruency of three angles
$\therefore \dfrac{x}{b} = \dfrac{b}{c}$	Measure of corresponding sides of Similar triangle are proportional
or $x = \dfrac{b^2}{c}$ (i)	
Again in $\Delta BDC \leftrightarrow \Delta BCA$	figure ii (b) and (i)
$\angle B \cong \angle B$	Common self congruent
$\angle BDC \cong \angle BCA$	Construction given, both measures 90
$\angle C \cong \angle A$	$\angle C$ and $\angle A$ complements of $\angle B$
$\therefore \Delta BDC \sim \Delta BCA$	Congruency of three angles.
$\therefore \dfrac{y}{a} = \dfrac{a}{c}$	Sides of similar triangles are proportional.
or $y = \dfrac{a^2}{c}$ (ii)	
But $y + x = c$	Supposition.
$\therefore \dfrac{a^2}{c} + \dfrac{b^2}{c} = c$	By (i) and (ii)
or $a^2 + b^2 = c^2$	
or $c^2 + a^2 + b^2$	Multiplying both sides with c.

Theorem 7a

Converse of Pythagoras Theorem

In a triangle if the sum of the squares of the measures of two sides is equal to the square of the measure of the third side, the triangles is a right angled triangle.



Given:

In a ΔABC, $m\overline{AB} = c$, $m\overline{BC} = a$, $m\overline{AC} = b$ such that $a^2 + b^2 = c^2$

To Prove:

$m\angle ACB = 90$ i.e. ΔACB is a right angled triangle.

Construction:

Draw $\overline{CD}$ perpendicular to $\overline{BC}$ such that $\overline{CD} \cong \overline{CA}$. Join the points B and D.

Proof:

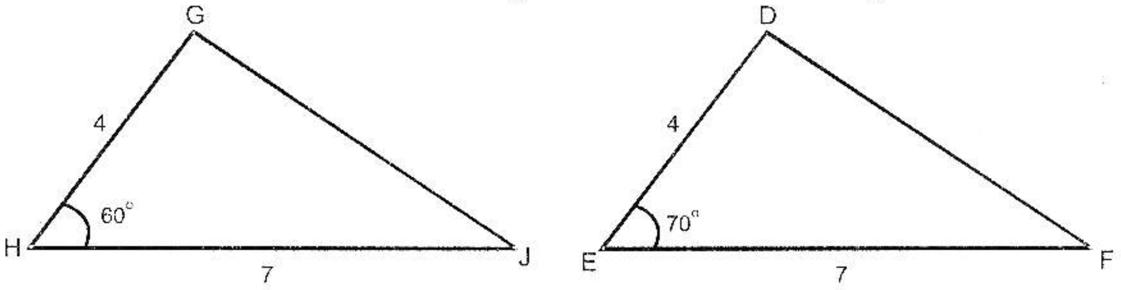
Statements	Reasons
ΔDCB is a right angled triangle	Construction
$(m\overline{BD})^2 = a^2 + b^2$	Pythagoras theorem
But $a^2 + b^2 = c^2$	Given
$\therefore (m\overline{BD})^2 = c^2$	
or $m\overline{BD} = c$	Taking square root of both sides.
Now in	
$\Delta DCB \leftrightarrow \Delta ACB$	
$\overline{CD} \cong \overline{CA}$	Construction
$\overline{BC} \cong \overline{BC}$	Common
$\overline{DB} \cong \overline{AB}$	Each is equal to c
$\therefore \Delta DCB \cong \Delta ACB$	$SSS \cong SSS$
$\therefore \angle DCB \cong \angle ACB$	corresponding angles of congruent triangles.
But $m\angle DCB = 90^\circ$	Construction
$\therefore \angle ACB = 90^\circ$	
And the ΔACB is a right angled triangle.	

EXAMPLES

Q.1 Two sides of a triangle measure 10 cm and 15 cm. Which of the following measure is possible for the third side?

Ans. As the sum of lengths of two sides is always greater than third side so construction of triangle is possible of third side is taken as 20 cm.

Q.2 In ΔGHT and ΔDEF shown in figure which of the following is true?



(a) $m\overline{DF} > m\overline{GJ}$ (b) $m\overline{DF} < m\overline{GJ}$

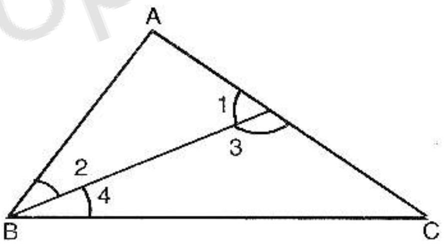
(c) $m\overline{DF} = m\overline{GJ}$ (d) $m\overline{DF} = 11$

Sol:

As we know that in a triangle, a side opposite to the greater angle is longer than the side opposite to the smaller angle. Hence statement (a) is true

Q.3 Prove that the difference of the measurers of two sides of a triangle less than the measure of third side.

Sol:



Given a ΔABC, $m\overline{AC} > m\overline{AB}$

To prove: $m\overline{BC} - m\overline{AC} < \overline{AB}$

Proof:

In ΔABC

As we know

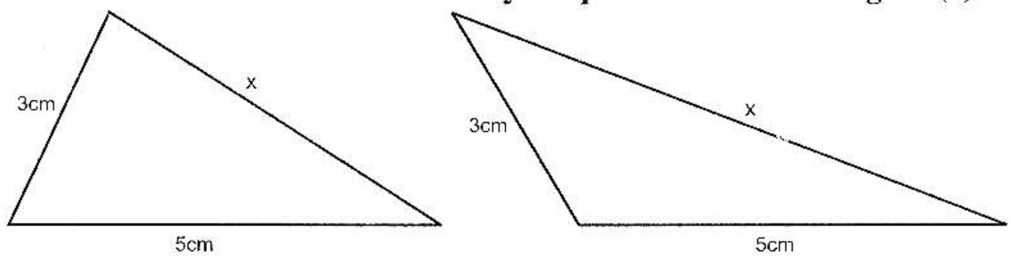
$m\overline{AB} + m\overline{AC} > m\overline{BC}$

or sum of the two sides of Δ is greater than third side

$m\overline{BC} - m\overline{AC} < m\overline{AB}$

$m\overline{BC} - m\overline{AC} < m\overline{AB}$ as required

Q.4 Two rods measure 3cm and 5cm. they are placed as shown in figure (a) and (b)



Find possible values of x to complete the triangle.

Sol: The possible value of x is

$5 + 3 > x$ and $5 - 3 < x$

$8 > x$ and $2 < x$

$\Rightarrow 2 < x < 8$

Because the Sum of measure of any two sides of a triangle is greater than the

measure of the third side i.e. $5 + 3 > x$ and difference of measure of two sides of triangle is less than the measure of the third side i.e. $5 - 3 < x$.

Q.5 In the ΔABC, $m < B = 75^0$ and $m < c = 55^0$ which of the sides of the triangle is longest and which the shortest.

Sol:

Given a ΔABC in which

$m\angle B = 75^0$

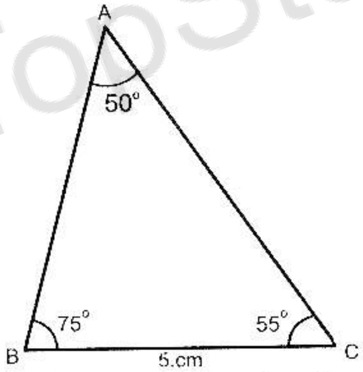
$m\angle C = 55^0$

As $m\angle A + m\angle B + m\angle C = 180^0$

$m\angle A + 75^0 + 55^0 = 180^0$

$m\angle A + 130^0 = 180^0$

$m\angle A = 50^0$



As we know in a triangle, the side opposite to greater angle is longer than the side opposite to smaller angle

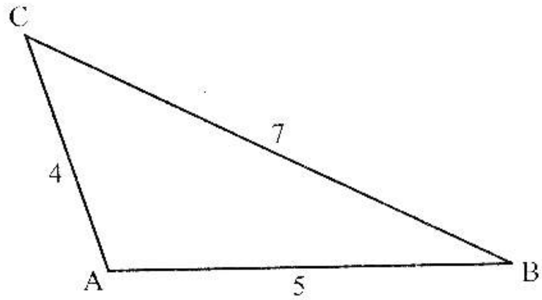
So $m\overline{AC} > m\overline{BC}$

Hence longest side is $\overline{AC}$

and shortest side is $\overline{BC}$

Q.6 In the ΔABC shown in figure find the biggest and smallest angles (write names only)

Sol:



Given: a ΔABC with

$m\overline{AB} = 5$, $m\overline{BC} = 7$, $m\overline{AC} = 4$

We want to find the biggest and smallest angles.

As we know in a triangle, the measure of the angle opposite to longer side is greater than that of the angle opposite to shortest side

so biggest angle is $m < BAC$

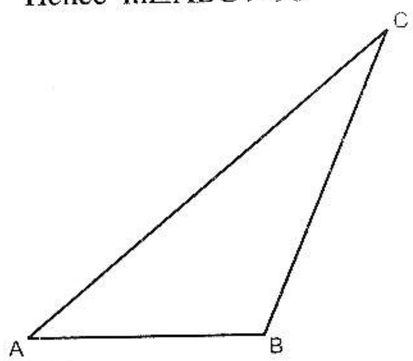
and Shortest angle is $m < ABC$

Q.7 In an obtuse angled triangle, the side opposite to the obtuse angle is longer than each of the other two sides.

Sol:

Given: Δ ABC is an obtuse angled triangle

Hence $m\angle ABC > 90^0$



To Prove:

$m\overline{AC} > m\overline{AB}$ and $m\overline{AC} > m\overline{BC}$

Since $\overline{AB}, \overline{BC}, \overline{AC}$ are the sides of triangle ABC. So

$m\overline{AC}+m\overline{AB}>m\overline{BC}$

Proof:

As m<ABC is an obtuse angle so it is the largest angle of ΔABC

Hence $m\angle B > m\angle A$ and also $m\angle B > m\angle C$

As in a triangle, the side opposite to greater angle is longer than the side opposite to the smaller angle.

So $m\overline{AC} > m\overline{AB}$ and $m\overline{AC} > m\overline{BC}$

Q.8 Prove that in a right angled triangle, the hypotenuse is longer than each of the other two sides.

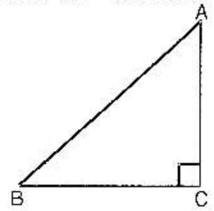
Sol:

Given:

ΔABC is a right angled triangle.

Hence $\overline{AB}$ is hypotenuse of ΔABC.

To Prove:



$m\overline{AB} > m\overline{AC}$

and $m\overline{AB} > m\overline{BC}$

Proof:

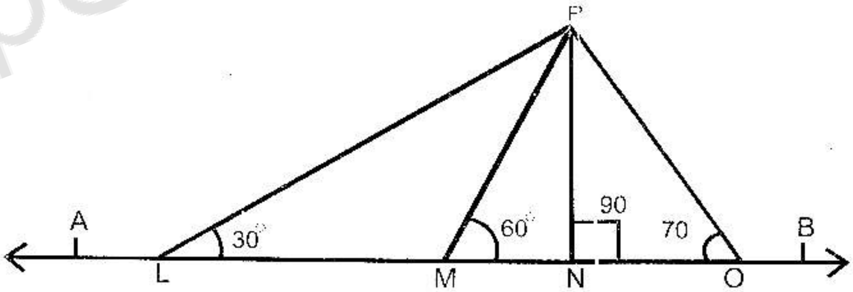
As ΔABC is a right angled triangle. So $m\angle C = 90^0$ is the largest angle and the remaining angles $\angle A$ and $\angle B$ are acute.

So $m\angle C > m\angle A$ and $m\angle C > m\angle B$

As the side opposite to the greater angle is longer than the side opposite to the smaller angle.

Hence $m\overline{AB} > m\overline{AC}$ and $m\overline{AB} > m\overline{BC}$

Q.9 · In the figure, P is any point and $\overleftrightarrow{AB}$ is a line. Which of the following is the shortest distance between the point P and the line $\overleftrightarrow{AB}$.



(a) (a) $m\overline{PL}$ (b) $m\overline{PM}$ (c) $m\overline{PN}$ (d) $m\overline{PO}$

Sol: As we know that from a point outside a line, the perpendicular is the shortest distance from the point to the line

As $m\overline{PN} \perp \overleftrightarrow{AB}$

So. $m\overline{PN}$ is the shortest distance.

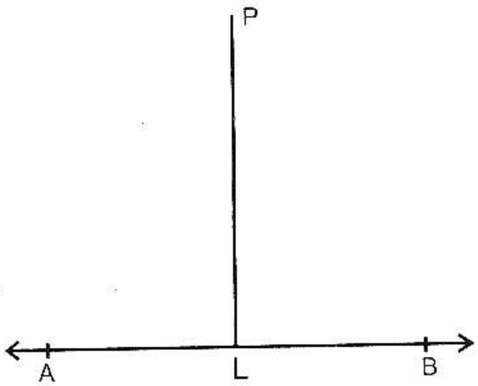
Q.10 In the figure, P is any point lying away from the line $\overleftrightarrow{AB}$. Then $m\overline{PL}$ will be the shortest distance if

(a) $m\angle PLA = 80^{\circ}$

(b) $m\angle PLB = 100^{\circ}$

(c) $m\angle PLA = 90^{\circ}$

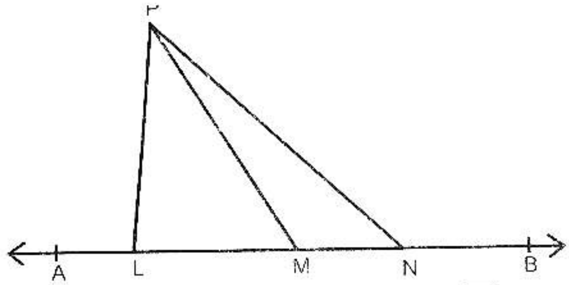
(d) $m\angle PLA = 70^{\circ}$



Sol: As we know that for a point outside a line, the shortest distance from a point to the line is perpendicular to line $\overleftrightarrow{AB}$. So

$m\angle PLA = 90^0$

Q.11 In the figure, $\overline{PL}$ is perpendicular to the line $\overleftrightarrow{AB}$ and $m\overline{LN} > m\overline{LM}$ prove that $m\overline{PN} > m\overline{PM}$



Sol: Here it is given $m\overline{PL}$ is perpendicular to line $\overleftrightarrow{AB}$ and $m\overline{LN} > m\overline{LM}$

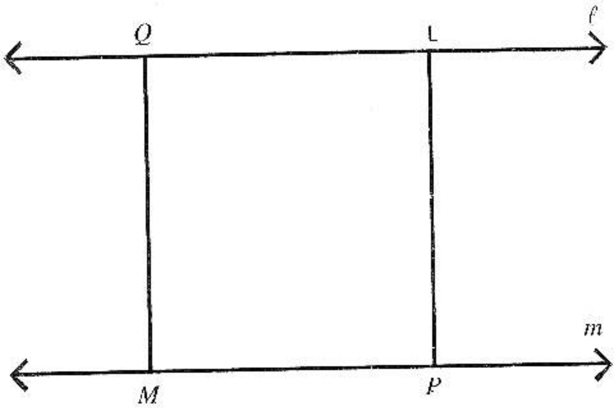
Proof:

Here $m\overline{PN} > m\overline{PM}$

As $\overline{PL}$ is the shortest distance from P to line $\overleftrightarrow{AB}$. So $\overline{PL} \perp \overleftrightarrow{AB}$

As we go away from point L, the distance from points to L increases Hence $m\overline{PN} > m\overline{PM}$

Q. 12 If l and m are two parallel lines in the figure then show that if $m\overline{PL}$ *and* $\overline{QM}$ are shortest distance between the lines l and m then $\overline{PL} \parallel \overline{MQ}$



Sol:

Given: l and m are two parallel lines and $\overline{PL}$ *and* $\overline{MQ}$ are shortest distances between lines l and m

To Prove: $\overline{PL} \parallel \overline{MQ}$

Proof: As $\overline{PL}$ is shortest distance between l and m.

So $\overline{PL} \perp l$

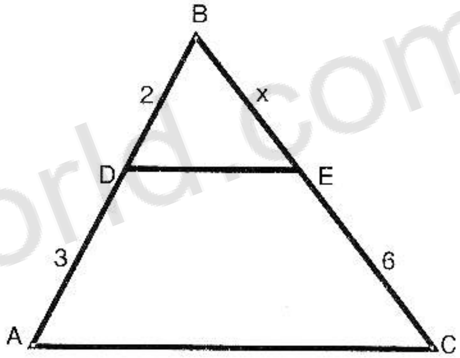
and $\overline{MQ}$ is also shortest distance between l and m.

So $PL \perp \overline{MQ}$ being perpendicular to a single line l should be parallel to each other

Hence $\overline{PL} \parallel \overline{MQ}$

Q.13 In a ΔABC if $\overline{DE} \parallel \overline{AC}$ as shown in the figure then x will be equal to

(a) 2
(b) 6
(c) 3
(d) 4



Sol: Given a ΔABC as shown $\overline{DE} \parallel \overline{AC}$

As we know in a triangle, a line parallel to one side of triangle and intersecting other two sides divides them proportionally.

So $\dfrac{m\overline{BE}}{m\overline{EC}} = \dfrac{m\overline{BD}}{m\overline{DA}}$.

$$\frac{x}{6} = \frac{2}{3}$$

$$\Rightarrow \quad 3x = 12$$

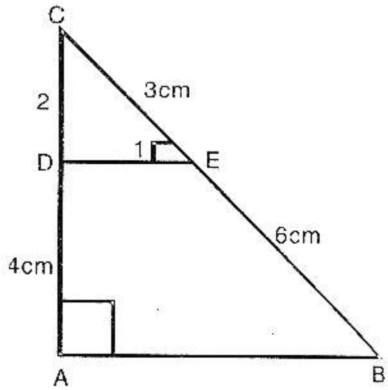
$$x = 4$$

Hence x = 4

Q.14 In the figure ΔABC is a right angled triangle with m$\angle$A = 90^0 and $\overline{DE}$ intersects the sides $\overline{AC}$ *and* $\overline{BC}$ at D and E respectively.

m $\angle$l will be equal to

(a) m$\angle$C
(b) m$\angle$B
(c) m$\angle$A
(d) m$\angle$D



Sol: **Given:** A ΔABC in which

$m\overline{CE} = 3cm$, $m\overline{EB} = 6cm$

$m\overline{CD} = 2cm$, $m\overline{DA} = 4cm$

As $\dfrac{m\overline{CE}}{m\overline{EB}} = \dfrac{3}{6} = \dfrac{1}{2}$

and $\frac{m\overline{CD}}{m\overline{DA}} = \frac{2}{4} = \frac{1}{2}$

So $\frac{m\overline{CE}}{m\overline{EB}} = \frac{m\overline{CD}}{m\overline{DA}}$

Hence $\overline{DE}$ intersects the two sides of Δ in same ratio.

So $\overline{DE} \parallel \overline{AB}$

Hence $m\angle 1 = m\angle B$ (corresponding angles)

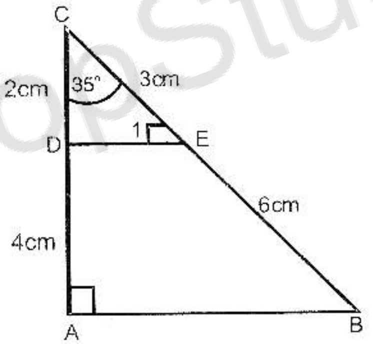
$(m\angle E \cong m\angle B)$

Q.15 In above figure if $m\angle C = 35^0$ then find

(i) $m\angle CED$

(ii) $m\angle DEB$

(iii) $m\angle ABC$



Sol: **(i)** Given a ΔABC as shown

Here $m\angle C = 35^0$

So $m\angle C + m\angle D + m\angle E = 180^0$

Or $35^0 + 90^0 + m\angle E = 180^0$

$125^0 + m\angle E = 180^0$

$m\angle E = 180^0 - 125^0$

or $m\angle CED = 55^0$

(ii) As $m\angle 1 + m\angle DEB = 180^0$ (supplementary angles)

or $m\angle CED + m\angle DEB = 180^0$

$55^0 + m\angle DEB = 180^0$

$m\angle DEB = 125^0$

(iii) As $\overline{DE} \parallel \overline{AB}$

So $m\angle CED = m\angle B$ (corresponding angles)

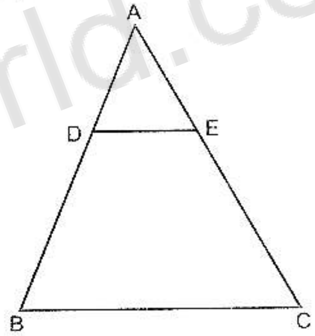
Or $m\angle B = m\angle CED$

$m\angle ABC = 55^0$

Q.16 In an equilateral triangle ABC shown in figure such that

$m\overline{AE} : m\overline{AC} = m\overline{AD} : m\overline{AB}$

Find all three angles of ΔADE and name it also



Sol: Given: ABC is an equilateral triangle DE is a line such that

$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$

We want to find $\angle A$, $\angle D$ and $\angle E$.

As $\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$

So $\overline{DE} \parallel \overline{BC}$

As ABC is an equilateral triangle.

So $m\angle A = m\angle B = m\angle C = 60^0$

As $m\angle E = m\angle C$

(corresponding angles)

and $m\angle D = m\angle B$

(corresponding angles)

So $m\angle D = 60^0$

$m\angle E = 60^0$

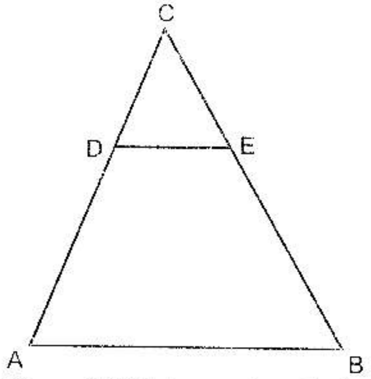
and $m\angle A = 60^0$

Thus ΔADE is an equilateral triangle

Q.17 In ΔABC,

$\overline{DE} \parallel \overline{AB}$ *and* $m\overline{AD}=3$, $m\overline{DC}=2$, $m\overline{BE}=6$ then $m\overline{EC}$ is equal to

(a) 8 (b) 4
(c) 5 (d) 9



Sol: Here ABC is a triangle in which $\overline{DE} \parallel \overline{AB}$ *and* $m\overline{AD}=3$, $m\overline{DC}=2$, $m\overline{BE}=6$

As $\overline{DE} \parallel \overline{AB}$

So DE divides AC and BC proportionally

i.e, $\dfrac{m\overline{BE}}{m\overline{EC}} = \dfrac{m\overline{AD}}{m\overline{DC}}$

$\dfrac{6}{m\overline{EC}} = \dfrac{3}{2}$

$\Rightarrow \quad \dfrac{m\overline{EC}}{6} = \dfrac{2}{3}$

$m\overline{EC} = \dfrac{2}{3} \times 6$

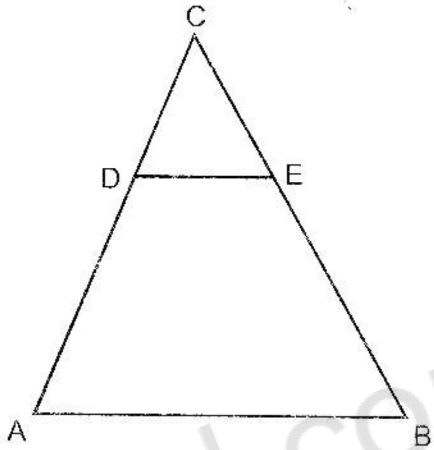
$= \dfrac{12}{3}$

$= 4$

So $m\overline{EC} = 4$

Q.18 In the figure. $m\overline{BC}$ is equal to

(a) 8 (b) 5
(c) 10 (d) 2



Sol: Here: $\overline{DE} \parallel \overline{AB}$

$m\overline{AD} = 3$

$m\overline{DC} = 2$

$m\overline{BE} = 6$

and $m\overline{EC} = 4$

Now as

$m\overline{BC} = m\overline{BE} + m\overline{EC}$

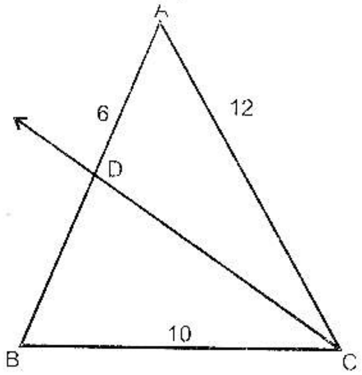
$= 6+4$

$= 10$

So $m\overline{BC} = 10$

Q.19 In ΔABC, $\overrightarrow{CD}$ bisects $\angle C$ and meets $\overline{AB}$ at D, $m\overline{BD}$ is equal to

(a) 5 (b) 16
(c) 10 (d) 18



Sol: Consider Δ ABC as shown. Here CD bisects $\angle C$ and meets $\overline{AB}$ at D.

As $\overrightarrow{CD}$ is the internal bisector of $\angle C$.

$$\text{So } \frac{m\overline{BD}}{m\overline{DA}} = \frac{m\overline{BC}}{m\overline{CA}}$$

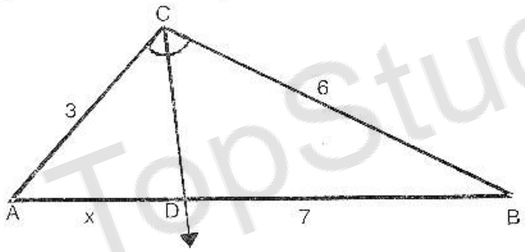
$$\frac{m\overline{BD}}{6} = \frac{10}{12}$$

$$m\overline{BD} = \frac{10}{12} \times 6$$

$$= \frac{60}{12}$$

$$m\overline{BD} = 5$$

Q.20 In ΔABC, $\overrightarrow{CD}$ bisects $\angle C$ if



$m\overline{AC} = 3cm$, $m\overline{BC} = 6cm$

And $m\overline{AB} = 7$ cm

Find $m\overline{AD}$ *and* $m\overline{DB}$

Sol: Consider Δ ABC, where $\overrightarrow{CD}$ bisects $\angle C$

and

$m\overline{AC} = 3cm$, $m\overline{CB} = 6cm$, $m\overline{AB} = 7cm$

$m\overline{AD} = ?$, $m\overline{DB} = ?$

Let $\overline{AD} = x$ then $\overline{DB} = 7 - x$

$$\text{As } \frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{CB}}$$

$$\frac{x}{7-x} = \frac{3}{6}$$

$$6x = 21 - 3x$$

$$6x + 3x = 21$$

$$9x = 21$$

$$x = \frac{21}{9} = \frac{7}{3}$$

Hence $m\overline{AD} = \frac{7}{3}$ cm

and $m\overline{DB} = 7 - x$

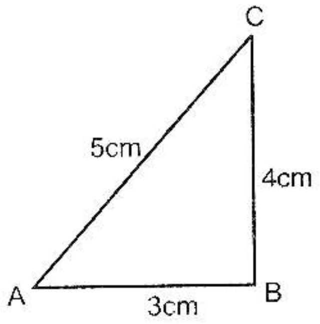
$$= 7 - \frac{7}{3}$$

$$= \frac{21-7}{3}$$

$$= \frac{14}{3} \text{ cm}$$

Q.21 The measure of sides of a triangle are 3cm, 4cm and 5cm, respectively. Prove that it is a right angled triangle.

Sol: Let ABC is a triangle with measure of sides as $\overline{AB} = 3$cm, $\overline{BC} = 4$cm, $\overline{CA} = 5$cm.



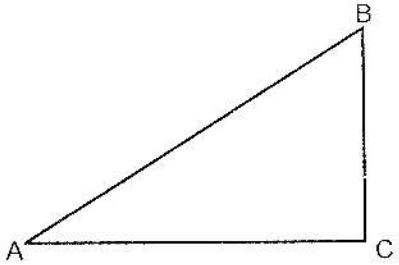
Here $\left|\overline{CA}\right|^2 = (5)^2 = 25$

and $\left|\overline{AB}\right|^2 + \left|\overline{BC}\right|^2$ $3^2 + 4^2 = 9 + 16 = 25$

since the square of length of one side is equal to the sum of squares of the lengths of other two sides. So by Pythagoras theorem, it is a right angled triangle with hypotenuse of measure 5cm.

Q.22 Verify that $a^2 + b^2$, $a^2 - b^2$ and 2ab are the measures of the sides of a right angled triangle where a and b are any two real numbers (a > b).

Sol:



Let $\overline{AB} = a^2 + b^2$

$\overline{BC} = a^2 - b^2$

and $\overline{AC} = 2ab$

Now $\left|\overline{AB}\right|^2 = (a^2 + b^2)^2$

$= a^4 + b^4 + 2a^2b^2$

$\left|\overline{AC}\right|^2 + \left|\overline{BC}\right|^2 = (2ab)^2 + (a^2 - b^2)^2$

$= 4a^2b^2 + a^4 + b^4 - 2a^2b^2$

$= a^4 + b^4 + 2a^2b^2$

$= (a^2 + b^2)^2$

so we have

$\left|\overline{AB}\right|^2 = \left|\overline{AC}\right|^2 + \left|\overline{CB}\right|^2$

Hence ABC is a right angled triangle so it is verified that $a^2 + b^2$, $a^2 - b^2$ and 2ab are the measures of the sides of a right angled triangle.

Q.23 The three sides of a triangle are of measure 5, x, 13 respectively. For what value of x, it will become base of a right angled triangle.

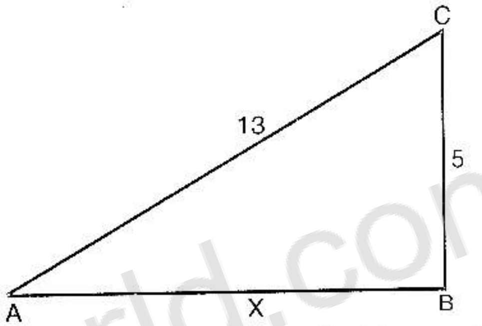
Sol:

Consider a right angled triangle

With $\overline{AB} = x$

$\overline{BC} = 5$

and $\overline{AC} = 13$



If x is the base of right angle ΔABC then we know by Pythagor theorem that

$(hyp)^2 = (Base)^2 + (perp)^2$

$(13)^2 = x^2 + (5)^2$

$169 = x^2 + 25$

$x^2 + 25 = 169$

$x^2 = 169 - 25$

$x^2 = 144$

$x = 12$

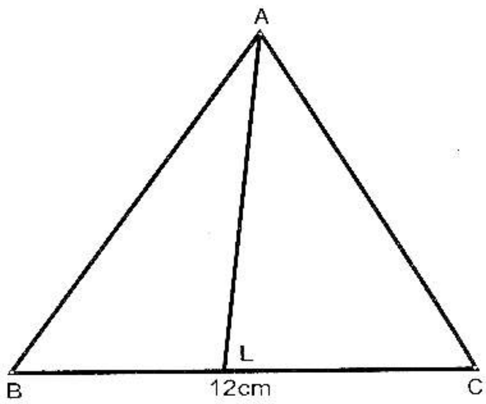
As x is measure of side

So x = 12 units

Q.24 In an equilateral triangle, t measure of one side is 12cm. find t measure of its altitude.

Sol: Consider an equilateral triangle AB

here $\overline{BC} = 12$



As it is an equilateral triangle,

Let L be the midpoint of side $\overline{BC} = 12$

So $\overline{BL} = 6cm$

Also $\overline{AB} = \overline{BC} = \overline{AC}$

So $\overline{AB} = 12$

As $\overline{AL}$ is attitude of ΔABC.

Hence ΔBLA is a right angled triangle

So by Pythagoras theorem

$$\left|\overline{AB}\right|^2 = \left|\overline{BL}\right|^2 + \left|\overline{AL}\right|^2$$

$$(12)^2 = (6)^2 + \left(\overline{AL}\right)^2$$

$$36 + \left(\overline{AL}\right)^2 = 144$$

$$\left(\overline{AL}\right)^2 = 144 - 36$$

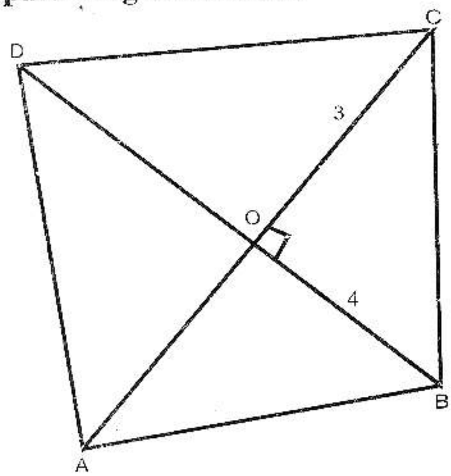
$$\left(\overline{AL}\right)^2 = 108$$

$$\left(\overline{AL}\right) = \sqrt{108}$$

$$= \sqrt{36 \times 3}$$

$$= 6\sqrt{3}cm$$

Q.25 Diagonals of parallelogram ABCD form four congruent triangles as shown in figure. Find the perimeter of the parallelogram ABCD.



Is a parallelogram ABCD forming four congruent triangles let O be the points of intersection of diagonals

Here

$$m\overline{OC} = 3 \quad and \quad m\overline{OB} = 4$$

As the four triangles are congruent so

$\overline{OC} = 3 \quad and \quad m\overline{OB} = 4$

In right angled ΔBOC

Applying Pythagoras theorem

$$\left|\overline{BC}\right|^2 = \left|\overline{BO}\right|^2 + \left|\overline{OC}\right|^2$$

$$= (4)^2 + (3)^2$$

$$= (16+9)$$

$$\left|\overline{BC}\right|^2 = 25$$

$$\Rightarrow \quad \left|\overline{BC}\right| = 5$$

as $\overline{AB} = \overline{BC} = \overline{CD} = \overline{AD}$

so perimeter of parallelogram ABCD is

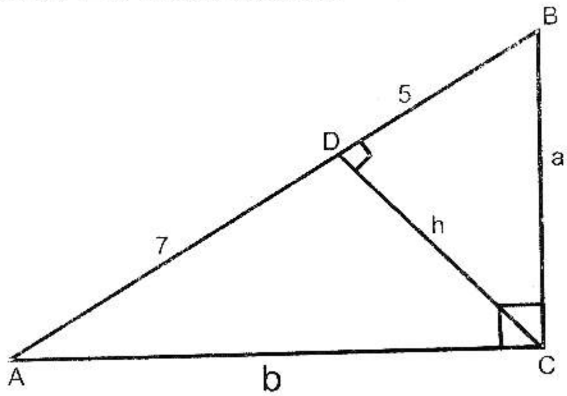
$$P = \left|\overline{AB}\right| + \left|\overline{BC}\right| + \left|\overline{CD}\right| + \left|\overline{AD}\right|$$

$$= 5 + 5 + 5 + 5$$

$$P = 20$$

So req perimeter is 20 units.

Q.26 In the ΔABC as shown in the figure, m$\angle$ ACB = 90^0 and $\overline{CD} \perp \overline{AB}$ Find the lengths of a, h and b if $m\overline{BD} = 5$ units and $m\overline{AD} = 7$ units



Sol: Given: a ΔABC as shown

$m\angle ACB = 90^0$

and $\overline{CD} \perp \overline{AB}$

to find a, h and b.

In right angled ΔBDC

$a^2 = 25 + h^2$ .... (i)

in right angled ΔADC

$b^2 = 49 + h^2$ .... (ii)

in right angled ΔABC

$a^2+b^2 = 144$ .... (iii)

adding (i) and (ii)

$a^2+b^2 = 74+2h^2$ .... (iv)

from (iii) and (iv)

$74 + 2h^2 = 144$

$2h^2 = 144-74$

$2h^2 = 70$

$h^2 = 35$

$h = \sqrt{35}$

Now we will find a and b

Put $h^2 = 35$ (in 1)

$a^2 = 25+35$

$a^2 = 60$

$a = \sqrt{60}$

$= \sqrt{4\times 15}$

$a = 2\sqrt{15}$

now put $h^2 = 35$ (in 2)

$b^2 = 49+35$

$b^2 = 48$

$b = \sqrt{84}$

$b = \sqrt{4\times 21}$

$b = 2\sqrt{21}$

So

$a = 2\sqrt{15}$

$h = \sqrt{35}$

$b = 2\sqrt{21}$

Q.27 Fill in the blanks

Sol: **The filled blanks are as:**

(i) Every line contains atleast _____ distinct points.

(ii) Every plane contains atleast ______ non-collinear points.

(iii) If two rays of two adjacent angles are opposite then angels are ___________

(iv) Two intersecting lines cannot be parallel to the same line is called ___.

(v) A triangle having all the three sides equal is called _______.

(vi) If two triangles are congruent then all the ______ sides and angles are congruent.

(vii) In similar triangles _________ are congruent.

(viii) The set of all points which lie outside a triangle is called ________ the triangle.

(ix) In any triangle sum of measures of its any two sides is always ________ side.

(x) From a point outside a line _________ is the shortest distance.

Answers:

(i) Two

(ii) Three

(iii) Supplementary

(iv) Play fair postulate

(v) Equilateral triangle

(vi) Three sides, three

(vii) All the three corresponding angles

(viii) Exterior

(ix) greater than third

(x) Perpendicular distance

Q.28 Write true or false

Sol: The true and false statements are separated as: Statements

(i) In a right angled Δ ABC, if $m\angle C = 90^\circ$. then $a^2 + b^2 = c^2$.

(ii) In a right angled Δ ACB, if $m\angle C = 90^\circ$, then $a^2 + b^2 = c^2$

(iii) A triangle whose sides are 3,4 and 5 is right angled triangle.

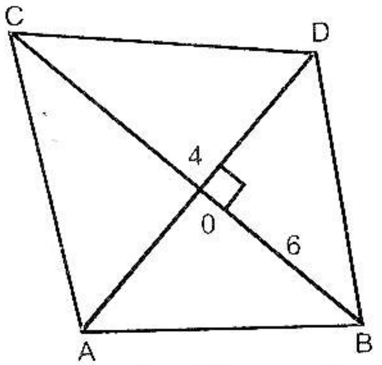
(iv) In a right angled triangle ABC in which $m\angle C = 90^\circ$, $\angle A$ and $\angle B$ are supplementary angles.

(v) If two triangles are similar, then their areas are always equal.

(i) True
(ii) False
(iii) True
(iv) False
(v) False

Q.29 A rhombus is shown in the figure Find the perimeter of ABCD

Sol: Given: A rhombus ABCD is shown in figure.
To find the perimeter of the ABCD.
As it is a rhombus
So $\overline{AB} = \overline{CD}$
& $\overline{AC} = \overline{BD}$



Let O be the point of intersecting of the diagonals AD and BC.
As being rhombus
$\overline{AD} \perp \overline{BC}$
so in right angled ΔBOD

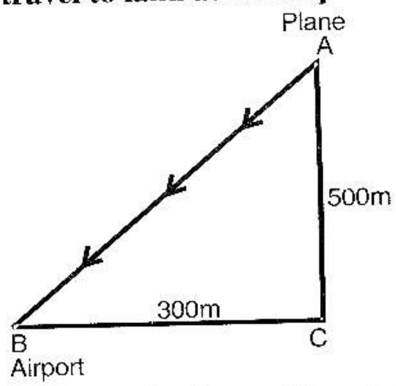
$$|\overline{BD}|^2 = |\overline{BO}|^2 + |\overline{OD}|^2$$
$$= (6)^2 + (4)^2$$
$$= 36 + 16$$
$$|\overline{BD}|^2 = 52$$
$$\overline{BD} = \sqrt{52}$$
$$\overline{BD} = \sqrt{4\times13}$$
$$\overline{BD} = 2\sqrt{13} \text{ units}$$

Now Perimeter of rhombus ABCD is

$$\text{Perimeter} = \overline{AB} + \overline{BD} + \overline{CD} + \overline{AC}$$
$$= \overline{BD} + \overline{BD} + \overline{BD} + \overline{BD}$$
$$= 4\overline{BD}$$
$$= 4(2\sqrt{13})$$
$$\text{Perimeter} = 8\sqrt{13} \text{ Units}$$

So required perimeter of rhombus ABCD is $8\sqrt{13}$ units

Q.30 A plane is at a height of 500 m and is 300m away from the airport as shown in figure. How much distance will it travel to land at the airport?



Sol: Here A be the position of plane and B be the position of airport.
s.t.

$\overline{AC} = 500m$

$\overline{BC} = 300m$

$\overline{AB} = ?$

Applying Pythagoras theorem on right angled triangle ABC

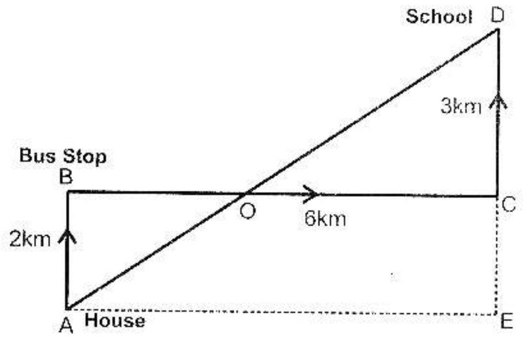
$$|\overline{AB}|^2 = |\overline{AC}|^2 + |\overline{BC}|^2$$
$$= (500)^2 + (300)^2$$
$$= 250000 + 90000$$
$$= 34000$$
$$|\overline{AB}|^2 = 34 \times 10000$$

so $|\overline{AB}| = \sqrt{34 \times 10000}$

$$= \sqrt{34}.100$$
$$= 100\sqrt{34}m$$

so required distance is $100\sqrt{34}m$

Q.31 Umer Khayyam travels to his school by the route as shown in the figure. Find $m\overline{AD}$. That is the direct distance from his house to school.



Sol: Here B be the bus stop and D be the School.

$m\overline{BC} = m\overline{AE}$

and $m\overline{AB} = m\overline{EC}$

so AED is right angled triangle by Pythagoras theorem

$$|\overline{AD}|^2 = |\overline{AE}|^2 + |\overline{ED}|^2$$
$$|\overline{AD}|^2 = (6)^2 + (5)^2$$
$$= 36 + 25$$
$$\overline{BD}^2 = 61$$
$$\overline{BD} = \sqrt{61}$$

So the direct distance from house to school is $\sqrt{61}$ km.

OBJECTIVE

Q.1 Four answers of each item are given from which only one is true. Tick the correct answer.

1. There are an infinite number of ________ on a line.
   (a) points (b) midpoints
   (c) rays (d) right bisector
2. Point is __________ term of geometry.
   (a) undefined (b) known
   (c) defined (d) unknown
3. Line is __________ term of Geometry.
   (a) defined (b) undefined
   (c) known (d) unknown
4. Plane is ____________ term of Geometry
   (a) defined (b) known
   (c) undefined (d) unknown
5. $\overleftrightarrow{AB}$ represents *(Lahore Board 2010)*
   (a) ray AB (b) line AB
   (c) segment AB (d) arc AB

6. $\overrightarrow{AB}$ represents.
   (a) line AB (b) ray AB
   (c) segment AB (d) minor arc AB

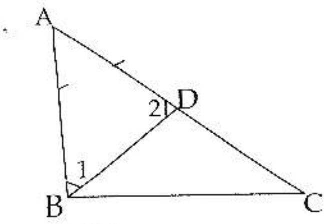
7. An exterior angle of a triangle is _____ every non-adjacent interior angle.
   (a) greater than (b) less than
   (c) zero (d) equal to

8. If two sides of a triangle are ____ in length, the measure of the angle opposite to the longer side is greater than that of the angle opposite to the shorter side.
   (a) zero (b) unequal
   (c) equal (d) parallel

9. If two angles of a triangle are unequal in measure the side opposite to the greater angle is the side opposite to the smaller angle.
   (a) longer than (b) shorter than
   (c) equal to (d) congruent to

10. Angles opposite to congruent sides are _______
   (a) supplementary (b) congruent
   (c) greater (d) shorter

11. In ΔABC, which of following is true



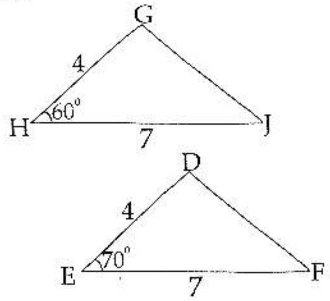
   (a) $m\overline{AC} > m\overline{AB}$
   (b) $m\overline{AB} > m\overline{AC}$
   (c) $m\overline{AC} = m\overline{AB}$
   (d) $m\overline{AD} = m\overline{AC}$

12. Sum of three angles in a triangle is
   (a) 360^0 (b) 270^0
   (c) 180^0 (d) 90^0

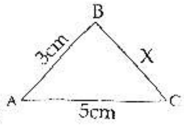
13. Two sides of a triangle measure 10cm and 15cm which of the following measure is possible for third side?
   (a) 5cm (b) 20cm
   (c) 25cm (d) 30cm

14. S. S. S $\cong$ ___________
   (a) S.S.S
   (b) S.A.S
   (c) A.A.S
   (d) A.A.A

15. In ΔGHJ and ΔDEF which of the following is true.



   (a) $\overline{DF} > \overline{GJ}$
   (b) $\overline{DF} < \overline{GJ}$
   (c) $\overline{DF} = \overline{GJ}$
   (d) $m\overline{DF} \parallel mGJ$

16. In ΔABC 

the value of x is_____.
   (a) x = 2
   (b) x > 2
   (c) x = 3
   (d) none of these

Note: The only possible value of x is $5 + 3 > x$ and $2 < x$ or $2 < x < 8$ because the sum of measures of any two sides of a triangle is greater than measure of third side i.e. $5 + 3 > x$ and difference of measure of two side of triangle is less than measure of third side i.e. $5 - 3 < x$

17. The difference of the measures of two sides of a triangle is _____than measure of the third side.
(a) greater than (b) less than
(c) equal to (d) congruent

18. In Δ ABC $m\angle B = 75^0$ and $m\angle C = 55^0$ which of the sides of triangle is longest?

(a) $\overline{AC}$ (b) $\overline{AB}$

(c) $\overline{BC}$ (d) All are congruent

Note: in ΔABC

$m\angle A + m\angle B + m\angle C = 180^0$

$m\angle A = 180^0 - 130^0$

$m\angle A = 50^0$

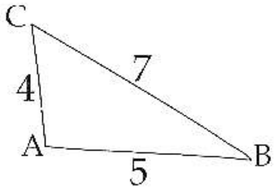
Longest side is $\overline{AC}$

19. In ΔABC, $m\angle B = 75^0$ and $m\angle C = 55^0$ which of the sides of triangle is shortest.

(a) $\overline{AC}$ (b) $\overline{AB}$

(c) $\overline{BC}$ (d) All are congruent

20. In ΔABC, $m\overline{AC} = 4$ and $\overline{BC} = 7$ and $m\overline{AB} = 5$,

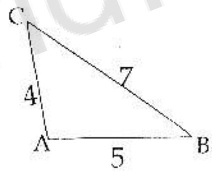


biggest angle is ________________.
(a) $m\angle ACB$ (b) $m\angle BCA$
(c) $m\angle CAB$ (d) $m\angle BCA$

21. In ΔABC, $m\overline{AC} = 4$,$m\overline{AB} = 5$ anc

$m\overline{BC} = 7$ then smallest angle i

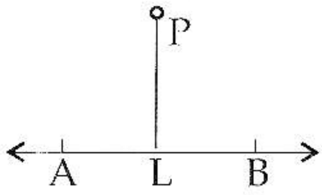
___________.



(a) $m\angle BAC$ (b) $m\angle ABC$
(c) $m\angle CAB$ (d) $m\angle BCA$

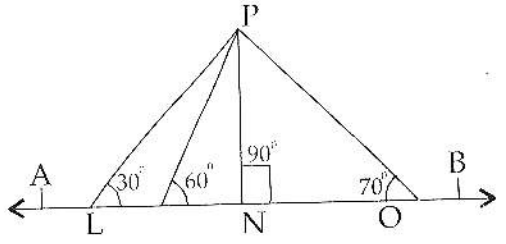
22. In a right angle triangle the hypotenuse is _____ each of the other two sides.
(a) longer than (b) shorter than
(c) equal to (d) perpendicular tc

23. In figure, P is any point lying away from the line $\overleftrightarrow{AB}$ then $m\overline{PL}$ will be shortest distance if



(a) $m\angle PLA = 80^0$
(b) $m\angle PLB = 110^0$
(c) $m\angle PLA = 90^0$
(d) $m\angle PLA = 70^0$

24. In figure, P is any point and $\overleftrightarrow{AB}$ is a line. Which of the following is shortest distance between P and the line $\overleftrightarrow{AB}$.



(a) $m\overline{PL}$ (b) $m\overline{PM}$

(c) $m\overline{PN}$ (d) $m\overline{PO}$

25. If a line segment intersects the two sides of a triangle in the same ratio, then it is ________the third side.

(a) parallel to

(b) perpendicular to

(c) less than (d) greater than

26. A line _______ to one side of a triangle and intersecting the other two sides divides them proportionally.

(a) perpendicular (b) parallel

(c) congruent (d) greater than

27. Area of triangle =______________.

(a) $\frac{1}{2}$ Base × height (b) $\frac{1}{2}\times\frac{\text{Base}}{\text{height}}$

(c) $\frac{1}{2}\times\frac{\text{height}}{\text{Base}}$ (d) $\frac{1}{2}$ Base × hypotenuse

28. If two intersecting lines form equal adjacent angles, the lines are ___.

(a) parallel (b) perpendicular

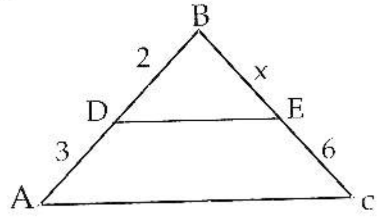
(c) collinear (d) congruent

29. A line segment has exactly _______ midpoint.

(a) one (b) two

(c) three (d) infinite

30. In a ΔABC if $\overline{DE} \parallel \overline{AC}$ then x will be equal to



(a) 2 (b) 6 (c) 3 (d) 4

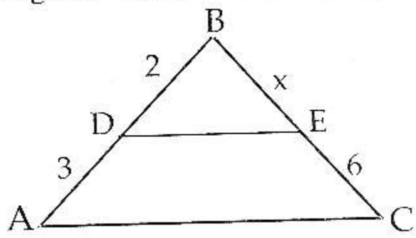
Note: $\overline{BE}:\overline{EC} = \overline{BD}:\overline{DA}$

$x : 6 = 2 : 3$

$3x = 12$

$x = \frac{12}{3} = 4$

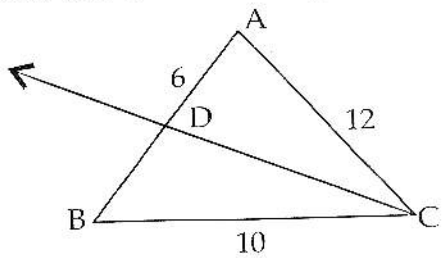
31. In figure, $m\overline{BC}$ = ___________.



(a) 2 (b) 10

(c) 5 (d) 8

32. In ΔABC, $\overline{CD}$ bisects $\angle C$ and meet $\overline{AB}$ at D then $m\overline{BD}$ = __.



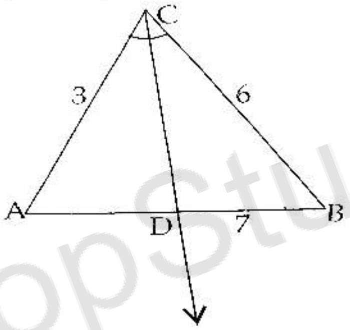
(a) 5 (b) 16
(c) 10 (d) 18

Note: $m\overline{BD} : m\overline{DA} = m\overline{BC} : m\overline{AC}$

$m\overline{BD} : 6 = 10 : 12$

$m\overline{BD} = \frac{6 \times 10}{12} = 5\text{cm}$

33. In ΔABC, $\overline{CD}$ bisects $\angle C$ if $m\overline{AC} = 3$, $m\overline{CB} = 6$, $m\overline{AB} = 7$ than $m\overline{AD}$ = __________.



(a) $\frac{7}{3}$ (b) $\frac{49}{6}$
(c) $\frac{6}{21}$ (d) $\frac{6}{49}$

Note: $\frac{m\overline{BD}}{m\overline{CB}} = \frac{m\overline{AD}}{m\overline{AC}}$

$\frac{7 - m\overline{AD}}{6} = \frac{m\overline{AD}}{3}$

$21 - 3\,m\overline{AD} = 6\,m\overline{AD}$

We know that

$$\left(\begin{array}{l} m\overline{AB} = m\overline{AD} + m\overline{BD} \\ 7 = m\overline{AD} + m\overline{BD} \\ m\overline{BD} = 7 - m\overline{AD} \end{array}\right)$$

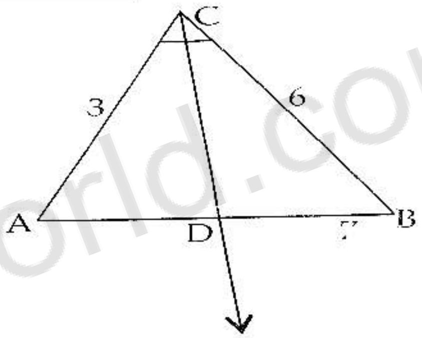
$21 = 6\,m\overline{AD} + 3\,m\overline{AD}$

$21 = 9\,m\overline{AD}$

$m\overline{AD} = \frac{21}{9} = \frac{7}{3}$ units

$m\overline{BD} = 7 - \frac{7}{3} = \frac{14}{3}$ units

34. In ΔABC, $\overline{CD}$ bisects $\angle C$ if $m\overline{AC} = 3$, $m\overline{CB} = 6$ and $m\overline{AD} = \frac{7}{3}$ then $m\overline{DB}$ = __________.



(a) $\frac{14}{3}$ (b) $\frac{21}{6}$ (c) $\frac{6}{21}$ (d) $\frac{6}{49}$

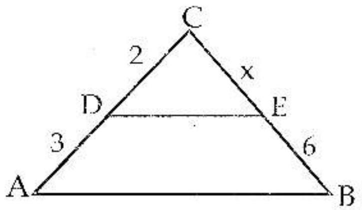
Note: $m\overline{AB} = m\overline{AD} + m\overline{DB}$

$7 = m\overline{AD} + m\overline{DB}$

$m\overline{DB} = 7 - m\overline{AD} = 7 - \frac{7}{3} = \frac{14}{3}$

35. In ΔABC, $\overline{DE} \parallel \overline{AB}$ and $m\overline{AD} = 3$, $m\overline{DC} = 2$ $m\overline{BE} = 6$ then $m\overline{CE}$ is equal to.

(a) 8 (b) 4
(c) 5 (d) 9



Note: $\frac{m\overline{CE}}{m\overline{EB}} = \frac{m\overline{CD}}{m\overline{DA}}$

$$\frac{x}{6} = \frac{2}{3}$$

$$x = \frac{2}{3} \times 6$$

$$m\overline{CE} = 4\text{cm}$$

36. If two triangles are ______, then measures of their corresponding sides are proportional.

(a) different (b) greater
(c) similar (d) scalene

37. In right angled triangle, the square of the length of ________ is equal to the sum of the squares of the lengths of the other two sides.

(a) base (b) perpendicular
(c) hypotenuse (d) altitude

38. In a triangle, if the sum of the squares of the measures of two sides is equal to the square of the measure of the third side, the triangle is.

(a) equilateral triangle
(b) isosceles triangle
(c) right angled triangle
(d) scalene triangle

39. Three sides of a triangle are of measure 5, x and 13 for what value of x it will become base of right angle triangle.

(a) 2 (b) 8
(c) 12 (d) 10

Note: $(H)^2 = (B)^2 + (P)^2$

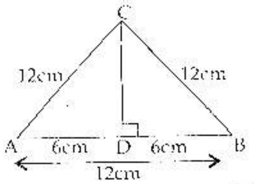
$(13)^2 = (x)^2 + (5)^2$

$169 = x^2 + 25$

$x^2 = 144$

$x = 12$

40. In an equilateral triangle, the measure of one side is 12cm, the measure of it altitude is.



(a) $6\sqrt{3}$ cm (b) $5\sqrt{3}$ cm
(c) $2\sqrt{3}$ cm (d) $\sqrt{3}$ cm

Note: $(BC)^2 = (BD)^2 + (DC)^2$

$(DC)^2 = (BC)^2 - (BD)^2$

$= (12)^2 - (6)^2 = 108$

altitude $m\overline{DC} = \sqrt{108} = 6\sqrt{3}$ cm

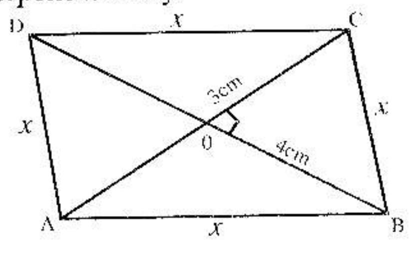
41. Diagonals of parallelogram ABCD form four congruent triangles as shown in figure, the perimeter of parallelogram ABCD is.

(a) 10 cm (b) 20 cm
(c) 5cm (d) 25cm

Note: Since ABCD is parallelogram

then $\overline{AC}$ and $\overline{BD}$ are its diagonal

Here $\overline{OC} = 3$cm, $\overline{OB} = 4$cm.
Since diagonal bisect each other, perpendicularly.



$\overline{OA} = 3$ and $\overline{OD} = 4$ (let BC = x)

By Pythagoras $x^2 = (3)^2 + (4)^2$

$x^2 = 25$

$x = 5$cm

Perimeter = $\overline{AB} + \overline{BC} + \overline{CD} + \overline{DA}$

$= x + x + x + x$

$= 4x$

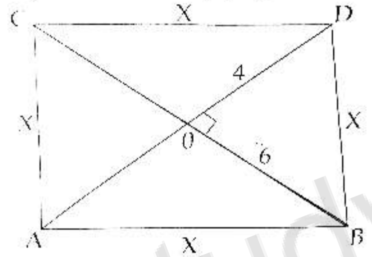
$= 4\ (5)$

$= 20$

42. A rhombus is shown in the figure, the perimeter of ABCD is.

(a) $8\sqrt{13}$ (b) $4\sqrt{13}$

(c) $2\sqrt{13}$ (d) $\sqrt{3}$



Note: Since ABCD is rhombus

here $\overline{OD} = 4$, $\overline{OB} = 6$

In ΔDOB

$x^2 = (4) + (6)^2$

$x^2 = 16 + 36$

$x^2 = 52$

$\Rightarrow \quad x = 2\sqrt{13}$

Rhombus's perimeter = $\overline{CA} + \overline{CD} + \overline{DB}$

$+ \overline{AB}$

$= x + x + x + x$

$= 4x$

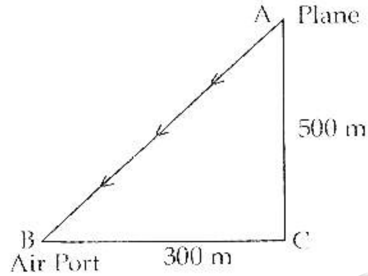
$= 4(2\sqrt{13})$

$= 8\sqrt{13}$

43. A plane is at a height of 500m and is 300 m away from the air port as shown in figure. How much distance will it travel to land at the air port.

(a) $100\sqrt{34}$ m (b) $10\sqrt{34}$ m

(c) $5\sqrt{34}$ m (d) $2\sqrt{34}$ m



Note: By Pythagoras theorem

$H^2 = P^2 + B^2$

$x^2 = (500)^2 + (300)^2$

$x^2 = \sqrt{340000}$

$x = 100\sqrt{34}$

44. In any triangle sum of measures of its any two sides is always ______ third side.

(a) smaller than third (b) greater than

(c) equal to (d) parallel

45. The set of all points which lie outside a triangle is called _____ of the triangle.

(a) interior (b) exterior

(c) angles (d) area

46. In similar triangles ______ are congruent.

(a) all the three corresponding angles

(b) all the three corresponding sides

(c) two sides (d) two angles

47. If two triangles are congruent then all the ___ sides and three angles are congruent.

(a) 2 (b) 3

(c) 1 (d) parallel

48. A triangle having all the three sides equal is called ____.

(a) isosceles triangle

(b) equilateral triangle

(c) right angled triangle (d) scalene triangle

49. Two intersecting lines cannot be parallel to the same line is called ________.

(a) play fair postulate
(b) angle-side-side postulate
(c) angle-side-angle postulate
(d) side-side-side postulate

50. If two rays of two adjacent angles are opposite then angles are __.

(a) complementary
(b) supplementary
(c) right angle
(d) acute angle

51. Every plane contains at least ___ non collinear points. (Lahore Board 2010)

(a) three (b) two
(c) one (d) infinite

52. Every line contains at least ____ distinct points. (Lahore Board 2010)

(a) three (b) two
(c) one (d) infinite

Answers

1	a	**2**	a	**3**	b	**4**	c
5	b	**6**	b	**7**	a	**8**	b
9	a	**10**	b	**11**	a	**12**	c
13	b	**14**	a	**15**	a	**16**	b
17	b	**18**	a	**19**	c	**20**	c
21	b	**22**	a	**23**	c	**24**	c
25	a	**26**	b	**27**	a	**28**	b
29	a	**30**	d	**31**	b	**32**	a
33	a	**34**	a	**35**	b	**36**	c
37	c	**38**	c	**39**	c	**40**	a
41	b	**42**	a	**43**	a	**44**	b
45	b	**46**	a	**47**	b	**48**	b
49	a	**50**	b	**51**	a	**52**	b